10. Controller Implementation

EN2142 Electronic Control Systems

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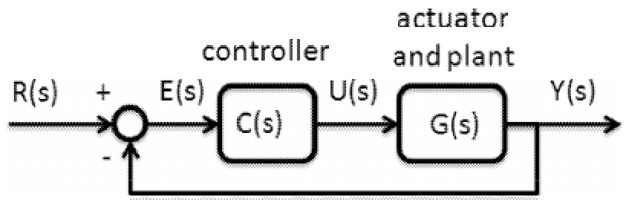
Controller Transfer Function

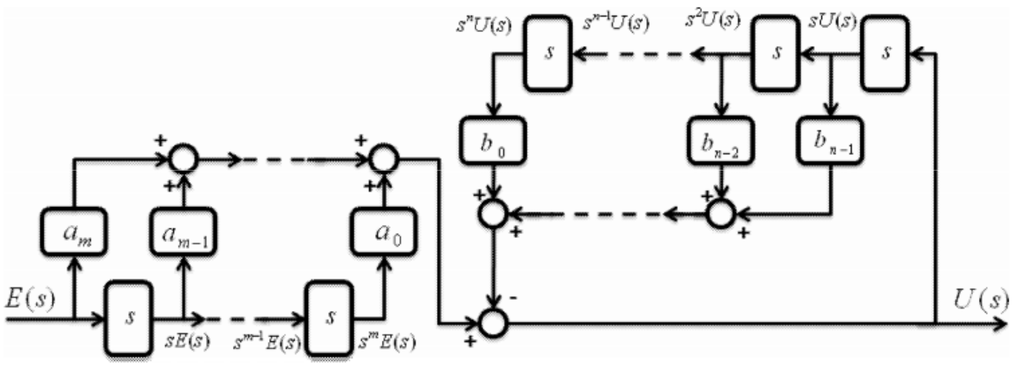
- Controller is a rational polynomial of *s*



$$C(s) = \frac{U(s)}{E(s)} = \frac{a_0 s^m + a_1 s^{m-1} + \dots + a_m}{b_0 s^n + b_1 s^{n-1} + \dots + 1}; n \geq m$$

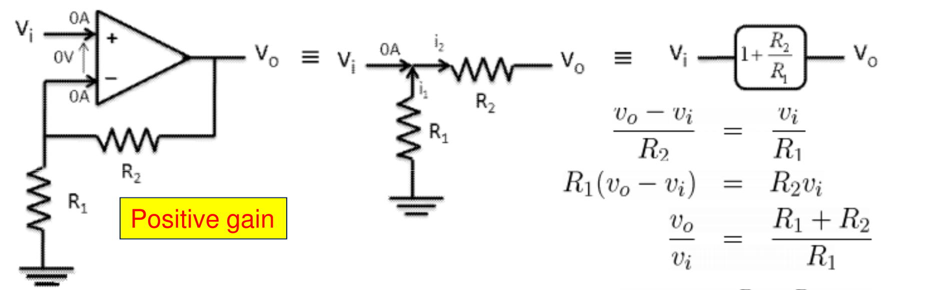
$$\begin{aligned} \{b_0 s^n + b_1 s^{n-1} + \ldots + b_{n-1} s + 1\} U(s) &= \{a_0 s^m + a_1 s^{m-1} + \ldots + a_m\} E(s) \\ U(s) &= \{a_0 s^m + a_1 s^{m-1} + \ldots + a_m\} E(s) \\ &\quad - \{b_0 s^n + b_1 s^{n-1} + \ldots + b_{n-1} s\} U(s) \end{aligned}$$

Controller Implementation (Analog)



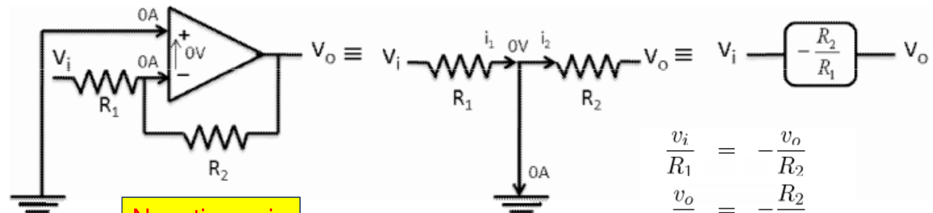
- How can the controller be physically implemented ?

OP Amps in Controller Design



Positive gain

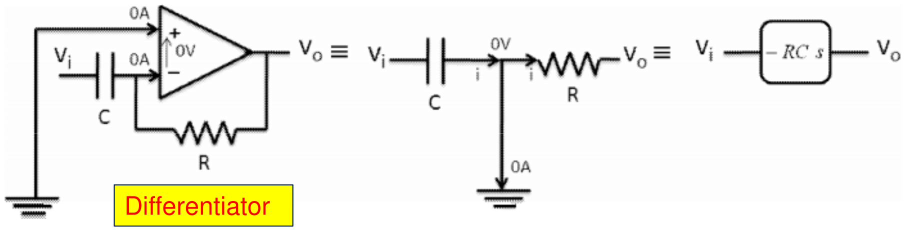
$$\begin{aligned} \frac{v_o - v_i}{R_2} &= \frac{v_i}{R_1} \\ R_1(v_o - v_i) &= R_2 v_i \\ \frac{v_o}{v_i} &= \frac{R_1 + R_2}{R_1} \\ V_o(s) &= \frac{R_1 + R_2}{R_1} V_i(s) \end{aligned}$$



Negative gain

$$\begin{aligned} \frac{v_i}{R_1} &= -\frac{v_o}{R_2} \\ \frac{v_o}{v_i} &= -\frac{R_2}{R_1} \\ V_o(s) &= -\frac{R_2}{R_1} V_i(s) \end{aligned}$$

OP Amps in Controller Design

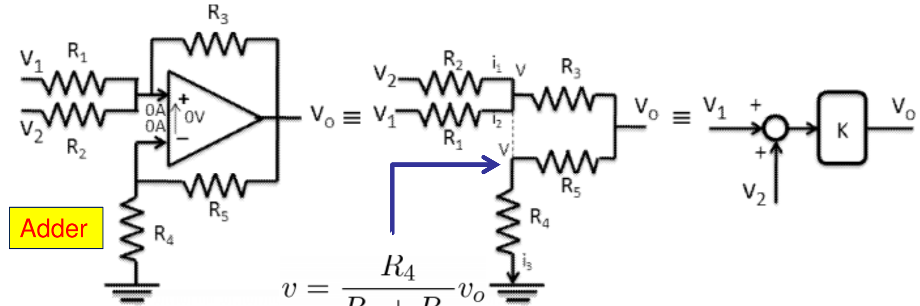


$$C\frac{dv_i(t)}{dt} + \frac{v_o(t)}{R} = 0$$

$$CsV_i(s) + \frac{1}{R}V_0(s) = 0$$

$$\frac{V_o(s)}{V_i(s)} = -RC.s$$

OP Amps in Controller Design



$$v = \frac{R_4}{R_4 + R_5}v_o$$

KCL

$$\frac{v_1 - v}{R_1} + \frac{v_2 - v}{R_2} + \frac{v_0 - v}{R_3} = 0$$

$$\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_o}{R_3} - \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)v = 0$$

$$\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_o}{R_3} - \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)\left(\frac{R_4}{R_4 + R_5}\right)v_o = 0$$

OP Amps in Controller Design

- Select $R_1 = R_2 = R_3 = R$

$$\frac{v_1 + v_2}{R} + \left(\frac{1}{R} - \frac{3}{R}\frac{R_4}{R_4 + R_5}\right)v_o = 0$$

- Further select $R_4 = R \text{ and } R_5 = 2R$

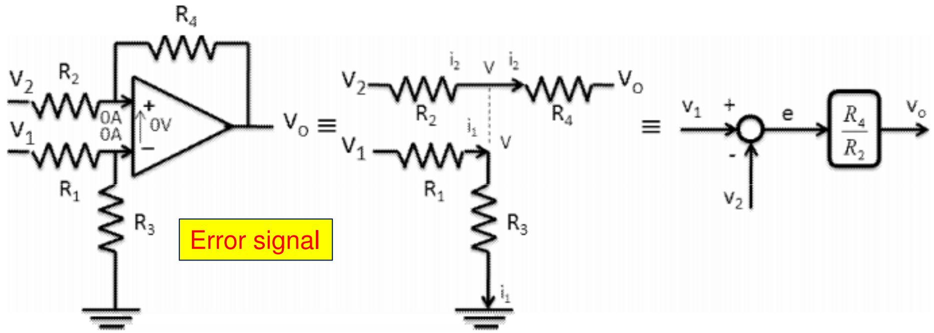
$$\frac{v_1 + v_2}{R} + \left(\frac{1}{R} - \frac{3}{R}\frac{2R}{3R}\right)v_o = 0$$

$$\frac{v_1 + v_2}{R} + \left(\frac{1}{R} - \frac{2}{R}\right)v_o = 0$$

$$v_1 + v_2 = v_0$$

$$V_1(s) + V_2(s) = V_o(s)$$

OP Amps in Controller Design



$$\frac{v_2 - v}{R_2} + \frac{v_o - v}{R_4} = 0$$

$$v = v_1\left(\frac{R_3}{R_1 + R_3}\right)$$

$$\frac{v_2}{R_2} + \frac{v_o}{R_4} - \left(\frac{1}{R_2} + \frac{1}{R_4}\right)v = 0$$

$$\frac{v_2}{R_2} + \frac{v_o}{R_4} - \left(\frac{1}{R_2} + \frac{1}{R_4}\right)\left(\frac{R_3}{R_1 + R_3}\right)v_1 = 0$$

Choose Resistors $\left(\frac{1}{R_2} + \frac{1}{R_4}\right)\left(\frac{R_3}{R_1+R_3}\right) = \frac{1}{R_2}$

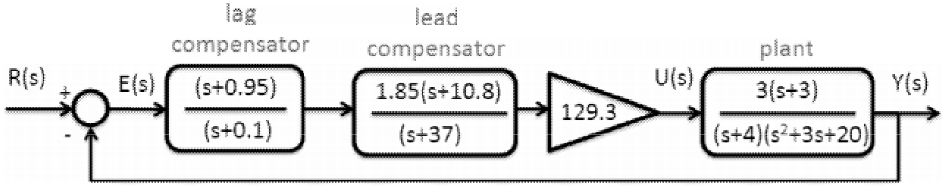
$$\frac{v_2}{R_2} + \frac{v_o}{R_4} - \frac{v_1}{R_2} = 0$$

$$v_o = \frac{R_4}{R_2}(v_1 - v_2)$$

$$V_o(s) = \frac{R_4}{R_2}[V_1(s) - V_2(s)]$$

Example: Analog Controller

- Implement the analog controller



- Controller transfer function

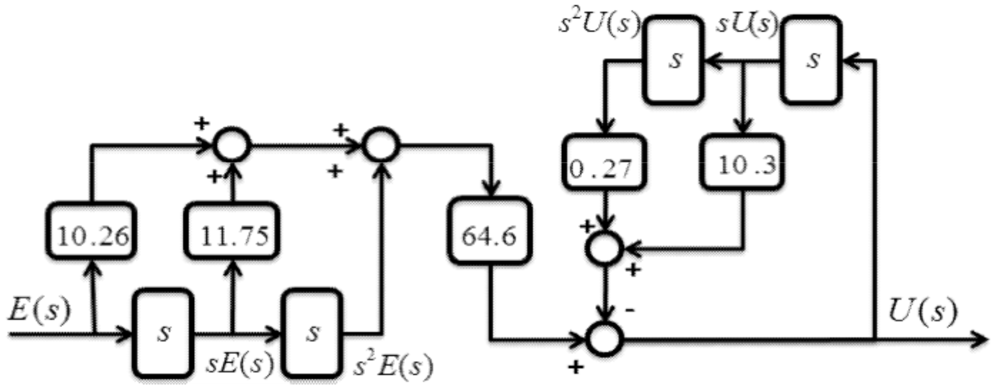
$$\frac{U(s)}{E(s)} = \frac{(s+0.95)}{(s+0.1)}1.85\frac{(s+10.8)}{(s+37)}129.3$$

$$= \frac{239.2(s^2+11.75s+10.26)}{s^2+37.1s+3.7}$$

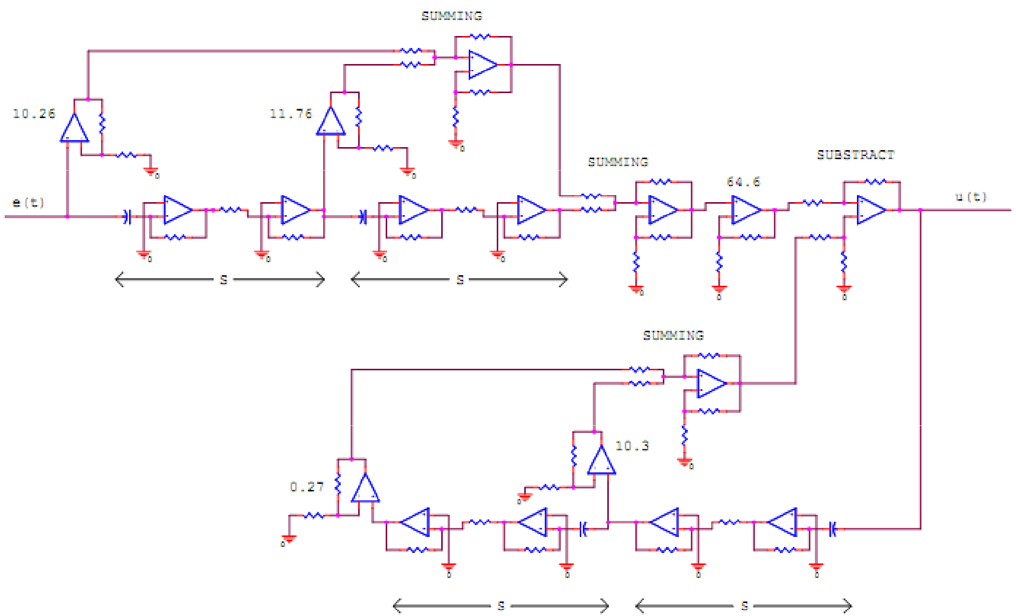
$$= \frac{64.6(s^2+11.75s+10.26)}{0.27s^2+10.03+1}$$

$$U(s) = 64.6[s^2+11.75s+10.26]E(s) - [0.27s^2+10.03s]U(s)$$

Example: Analog Controller

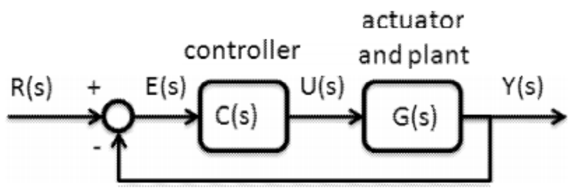


Example: Analog Controller

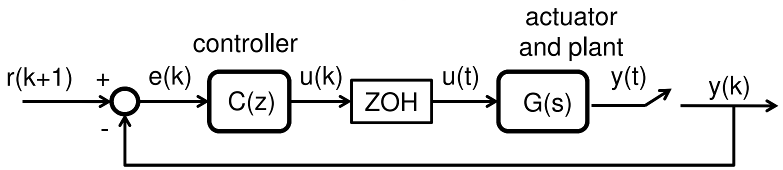


Digital Control : Digital Redesign

- Analog Controller => Discrete approximation

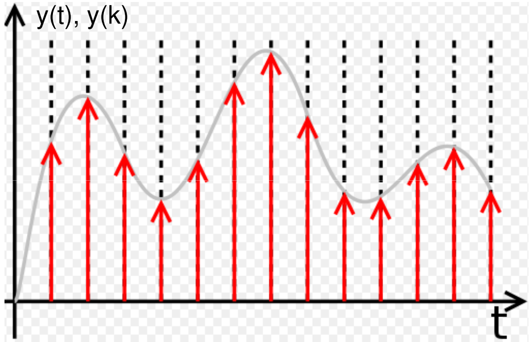


With a sampler and a zero order hold circuit

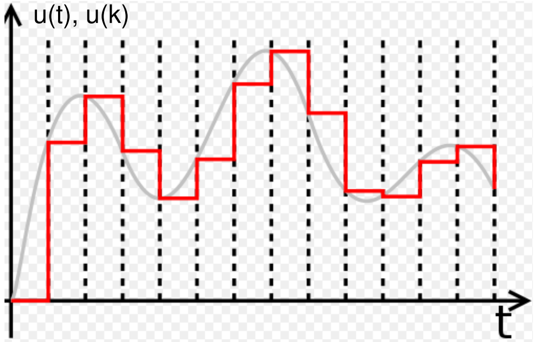


Sampler and ZoH

- **Sampler**
  Continuous response is sampled at T intervals. y(t) => y(k) then discrete controller can accept the feedback y(k)



- **Zero Order Hold**
  holds discrete control input for a period of T till next control input arrives. u(k) => u(t) then u(t) can actuate the continuous plant G(s)



Digital Redesign

- Analog Controller => Discrete approximation

$$C(s) = \frac{U(s)}{E(s)} = \frac{a_0 s^m + a_1 s^{m-1} + \dots + a_m}{b_0 s^n + b_1 s^{n-1} + \dots + 1}; n \geq m$$

$$b_0 \frac{d^n u(t)}{dt^n} + b_1 \frac{d^{n-1} u(t)}{dt^{n-1}} + \ldots + u(t) = a_0 \frac{d^m e(t)}{dt^m} + a_1 \frac{d^{m-1} e(t)}{dt^{m-1}} + \ldots + a_m e(t)$$

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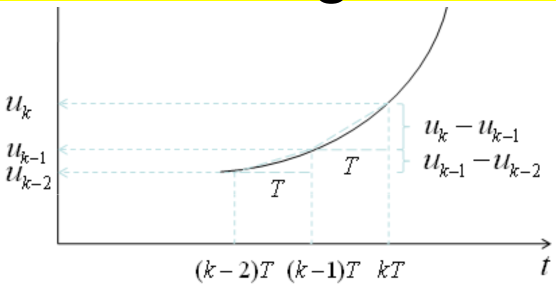
Discrete approximations for continuous variables

Digital Redesign



- Newton Backward difference method

$$u(t) \approx u_k \qquad e(t) \approx e_k$$

$$\frac{du(t)}{dt} \approx \frac{u_k - u_{k-1}}{T} \qquad \frac{de(t)}{dt} \approx \frac{e_k - e_{k-1}}{T}$$

$$\frac{du^2(t)}{dt^2} \approx \frac{u_k - 2u_{k-1} + u_{k-2}}{T^2} \qquad \frac{de^2(t)}{dt^2} \approx \frac{e_k - 2e_{k-1} + e_{k-2}}{T^2}$$

$$\vdots \qquad \vdots$$

$$f(u_{k-1}, u_{k-2}, \ldots, u_{k-n}) + u_k = g(e_k, e_{k-1}, \ldots, e_{k-m})$$

$$u_k = g(e_k, e_{k-1}, \ldots, e_{k-m}) - f(u_{k-1}, u_{k-2}, \ldots, u_{k-n})$$

Example : Digital Redesign

Digitally redesign the following analog controller

$$0.27 \frac{d^2 u(t)}{dt^2} + 10.3 \frac{du(t)}{dt} + u(t) = 64.6\left(\frac{d^2 e(t)}{dt^2} + 11.75 \frac{de(t)}{dt} + 10.26 e(t)\right)$$

Discrete approximation

$$0.27 \frac{u_k - 2u_{k-1} + u_{k-2}}{T^2} + 10.3 \frac{u_k - u_{k-1}}{T} + u_k = 64.6 \left(\frac{e_k - 2e_{k-1} + e_{k-2}}{T^2} + 11.75 \frac{e_k - e_{k-1}}{T} + 10.26 e_k \right)$$

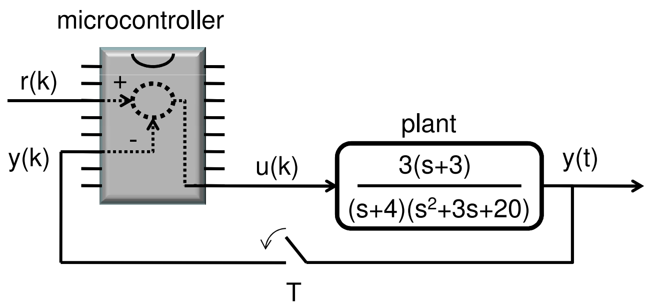
$$\left(\frac{0.27}{T^2} + \frac{10.3}{T} + 1\right) u_k = 64.6 \left\{ \left(\frac{1}{T^2} + \frac{11.75}{T} + 10.26\right) e_k - \left(\frac{2}{T^2} + \frac{11.75}{T}\right) e_{k-1} + \frac{1}{T^2} e_{k-2} \right.$$

$$- \left(\frac{0.54}{T^2} + \frac{10.3}{T}\right) u_{k-1} + \frac{0.27}{T^2} u_{k-2}$$

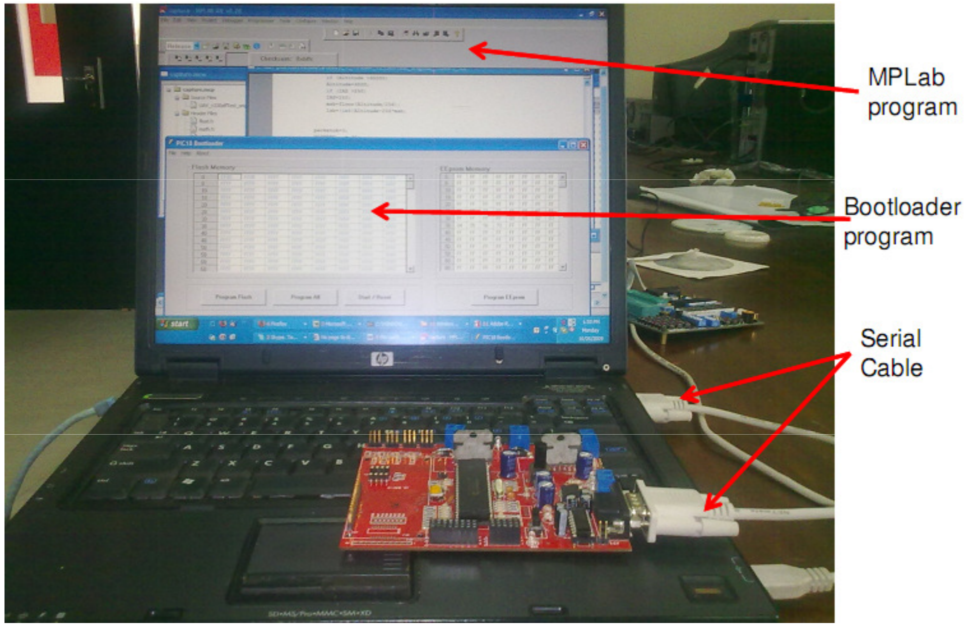
Example : Digital Redesign

$$
\begin{aligned}
(108+206+1)u_k-(216+206)u_{k-1} &= 64.6\{(400+235+10.26)e_k \\
+108u_{k-2} &\quad -(800+235)e_{k-1}+400e_{k-2}\} \\
315u_k-422u_{k-1}+108u_{k-2} &= 64.6\{645.26e_k-1035e_{k-1} \\
&\quad +400e_{k-2}\}
\end{aligned}
$$

$$u_k = 132.3e_k - 212.3e_{k-1} + 82e_{k-2} + 1.3u_{k-1} - 0.3u_{k-2}$$



Digital Controller Implementation



Analog- Digital Comparison

- **Digital control**
  - Easily programmable/reprogrammable in contrast to changing resisters/capacitors in analog controllers)
  - Easier to implement complex control systems
  - Can be integrated with remote systems through digital communication
  - Detailed user interfaces are available
  - Lower cost per controller
- **Analog control**
  - Simple (hardware only)
  - appropriate for mass produced devices
  - Fast feedback action (inner loop of control systems)
  - Reliable (only hardware failures possible)

Complex Control Systems

- Has many control loops, some of them are analog, and some others can be digital.
- Examples
  - Mars Rovers : 10-20 control loops
  - Aircrafts : 50+ control loops
  - Automobile : 5-20 control loops
- One controller can have digital and analog sections in it
- Controller parameters (gains) change dynamically to suit the operating conditions (**Adaptive Control**)
- Energy saving during operation can be done by critically damping control systems (**Optimal Control**)
- Energy critical applications such as space missions need to be designed with optimal controllers